NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2109

FREQUENCY RESPONSE OF POSITIVE-DISPLACEMENT VARIABLE-STROKE FUEL PUMP

By Harold Shames, Seymour C. Himmel
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SUMMARY

The dynamic characteristics of an axial-piston, variable-stroke jet-engine fuel pump were experimentally determined by frequency-response tests. It was found that the pump can be approximated by a first-order linear system with an average time constant of 0.04 second. The order of magnitude of the lag of the pump is negligible compared with current jet engines. It was also established that such a pump may be used for obtaining sinusoidal fuel pressures of variable frequency and amplitude.

INTRODUCTION

The exploration of the potentialities of jet-propulsion power plants during transient operation has indicated the possibility of utilizing experimental frequency-response techniques to obtain basic engine information. The frequency-response method, employing sinusoidal inputs to the engine and measuring the magnitude and the phase angle of the sinusoidal outputs, shows promise of becoming an effective means of supplementing the usual time-response method, which uses step inputs. A frequency-response test on a complete engine can yield information on each component of the engine with more accuracy and with less analysis than is required for a step-input, time-response test.

The use of frequency-response methods to study jet-engine-control combinations in the dynamic state leads to an analysis similar to that employed in linear electromechanical servomechanisms. Thus, any complex system is broken into simple components or blocks and the transfer function, a means of defining the time relation between output and input, is determined for each block. The characteristics of the entire system are then obtained by combining the blocks and their transfer functions (references 1 and 2).

A block diagram of a jet-engine configuration with an independent control (open loop) is shown in figure 1. This configuration may be changed into a closed-loop system by having the output variables control the input variable (reference 3). In any system, whether open or closed loop, the fuel pump serves as the link between the fuel control and the engine (with the fuel-distribution system considered as part of the engine) and consequently the pump dynamic characteristics are of prime importance to the over-all characteristics of the system.

An investigation of the dynamic characteristics of an axial-piston variable-stroke jet-engine fuel pump was therefore conducted at the NACA Lewis laboratory to:

1. Determine experimentally the transfer function of such a pump

2. Compare the transfer function of such a pump with that of current jet engines

3. Develop a fuel system that will produce a sinusoidal fuel pressure of variable frequency, amplitude, and base pressure to serve as the input to an engine for frequency-response tests

The variable-stroke axial-piston-type pump was chosen because of its wide application to jet engines and also because its steady-state characteristics showed a linear relation between fuel pressure and control oil pressure that was independent of the pump load or speed.

SYMBOLS

The following symbols are used in this report:

a	constant
f_c	corner frequency, (cycles/sec)
$G(j\omega)$	frequency-variant part of transfer function
j	$\sqrt{-1}$

K_1 constant relating steady-state change in fuel pressure to steady-state change in control oil pressure (frequency-invariant part of transfer function)

K_2 constant relating steady-state change in engine speed to steady-state change in fuel pressure (frequency-invariant part of transfer function)

$KG(j\omega)$ transfer function

k constant relating flow to pressure drop, ((lb/sq in.)/cu ft/sec)

$N(j\omega)$ sinusoidal component of engine speed

$P_c(j\omega)$ sinusoidal component of control oil pressure

$P_f(j\omega)$ sinusoidal component of fuel pressure

ΔP instantaneous pressure drop across restriction

Q instantaneous flow out of cylinder, (cu ft/sec)

Q_{max} maximum flow of fluid out of cylinder, (cu ft/sec)

$s = j\omega$

t time, (sec)

τ time constant, (sec)

ω angular frequency, (radian/sec)

Subscripts:

1 fuel pump

2 engine

c control oil pressure

ANALYSIS

The transfer function of a fuel pump is a means of defining the time relation between the change in output flow or pressure and the change in the input signal. The input signal to the pump investigated was a control oil pressure and the output was the fuel pressure. The pump transfer function is therefore

$$K_1G_1(j\omega) = \frac{P_f(j\omega)}{P_c(j\omega)} \tag{1}$$

where $G_1(j\omega)$ is the frequency-variant part of the pump transfer function. The block diagram of this pump in combination with a jet engine is shown in figure 1.

A jet engine will have its transfer function relating, for example, change in engine speed to change in fuel pressure in the dynamic state, as shown in figure 1 and reference 4. That is,

$$K_2G_2(j\omega) = \frac{N(j\omega)}{P_f(j\omega)} \tag{2}$$

where $K_2G_2(j\omega)$ is the engine transfer function and $G_2(j\omega)$ is the frequency-variant part of the transfer function.

The transfer function of the fuel pump and engine combination will then be

$$KG(j\omega) = K_1G_1(j\omega)\ K_2G_2(j\omega) \tag{3}$$

and the importance of the pump transfer function $K_1G_1(j\omega)$ on the system transfer function $KG(j\omega)$ will depend on the relative time response of $G_1(j\omega)$ and $G_2(j\omega)$.

APPARATUS

Description and Operation of Pump

The fuel pump, schematically shown in figure 2, is a variable-stroke multipiston pump with an internal servocontrol that automatically adjusts the outlet fuel pressure, as dictated by an external control oil pressure. This fuel pressure is a linear function of the control oil pressure.

The pumping pistons are driven by a wobble plate that is pivoted on the drive shaft. The angle of the wobble plate is determined by a control piston and plunger. The valving mechanism consists of an eccentrically mounted plate with circular ports that progressively vent the piston chambers to either the intake or exhaust annular concentric chambers. A built-in gear pump supplies oil at high pressure to move the control piston and plunger that actuate the wobble plate and to return the pumping pistons. The high-pressure oil is also used for lubrication and can be used, after throttling, as the external control oil applied to the servopiston.

Although the details given in the preceding sections are exclusive features of this particular pump, the basic mechanism of the wobble plate and control cylinder is used rather extensively to obtain variable stroke in axial-piston pumps. The principal lag in such a system is the time required to fill the control cylinder with fluid. Such a system may be characterized as a first-order lag with its particular time constant (reference 5). Thus, it is believed that the methods of this investigation are generally applicable for any pump of this type operating in the same pressure range, provided the controlling mechanism is linear.

Auxiliary Equipment

Frequency-response testing of electric and electronic equipment is a well-established practice and is well documented in literature. Frequency-response investigations have recently been conducted on hydraulic and mechanical systems such as control units, hydraulic transmissions, and so forth. (See references 5 and 6.) In these tests the inputs were always either electric signals or mechanical displacements, and the frequency band of interest was usually 5 to 50 cycles per second. In the case of electric and electronic equipment, the working frequencies are even higher.

Hydraulic Sine-Wave Generator

For this investigation, the input required was a hydraulic pressure and preliminary calculations indicated that the frequency band of interest would be 0 to 10 cycles per second. Hence, it was necessary to devise a hydraulic sine-wave generator of variable frequency and amplitude that would operate in the region of 0 to 10 cycles per second.

The principle of operation of the hydraulic sine-wave generator is to achieve a sinusoidal flow of the fluid in the system and to force this flow through a restriction to produce a sinusoidal pressure variation. The volume of flow must be kept sufficiently small to insure a practically linear relation between pressure and flow over the range of pressures used. A schematic drawing of a system employing this principle is shown in figure 3, where the sinusoidal flow is produced by an eccentrically mounted circular cam moving a piston, and the restriction is a needle valve.

The flow out of the cylinder can be written as

$$Q = Q_{max} \sin \omega t \tag{4}$$

If Q_{max} is sufficiently small and the flow is assumed laminar, the pressure drop across the restriction in the line can be written as

$$\Delta P \cong kQ \tag{5}$$

or, by substitution,

$$\Delta P \cong kQ_{max} \sin \omega t \tag{6}$$

which shows that the desired sinusoidal pressure variation is theoretically achieved. In the apparatus shown in figure 4, variable-frequency cam motion is obtained by having a constant-speed electric motor drive a variable-speed transmission to which is attached a cam consisting of an eccentrically mounted ball bearing. This cam acts on a spring-returned flat-foot follower that is attached to the piston rod of a stock hydraulic actuator cylinder. The exit ports of the cylinder are joined to produce a net displacement of fluid equal to the area of the piston rod times the eccentricity of the cam.

The sinusoidal flow is then directed to an accumulator through a needle valve, which serves as the variable restriction for amplitude control of the pressure pulse. The hydraulic accumulator is used both as a reservoir and as a means of loading the system to a base pressure. In this manner, a pressure pulse of variable frequency and amplitude and adjustable base pressure is available on the actuator side of the amplitude-control valve.

The phase shift between the piston position and the pressure wave, due to the accumulator-valve combination, did not affect the operation of the system because all measurements were referenced to the pressure wave.

PROCEDURE AND INSTRUMENTATION

A schematic diagram of the setup used for the frequency-response investigation of the pump is shown in figure 5. The sine-wave generator described in the preceding section was used to furnish the control oil pressure for the pump. Fuel was supplied to the pump from a tank by a booster pump. The pump discharged through a throttle valve back to the fuel tank. Fuel pressure was measured just ahead of the throttle valve by means of a strip-chart pressure recorder. Control oil pressure was measured just ahead of the inlet to the pump servo unit on another strip-chart pressure recorder. The pressure recorders were synchronized by solenoid-actuated timing pens that recorded 1/10-second intervals on the strip charts. Lubricating oil and charge oil were supplied to the pump from an oil tank; the oil pressure was generated by the internal gear pump built into the fuel pump.

Vibration of the test bed, caused by the pump drive system, necessitated using coils of tubing as mechanical filters in the lines leading to the pressure recorders in order to attenuate these vibrations, which were approximately 20 cycles per second. Equal lengths of tubing were inserted between the pump and the pressure recorders to obtain identical hydraulic circuits. A comparison of simultaneous recordings of the same sinusoidal pressure by both strip-chart recorders showed the two systems to be dynamically identical.

The following procedure was followed during the frequency-response tests:

The pump was brought up to the operating speed and the control oil pressure was adjusted by means of the helium pressure in the accumulator to give the predetermined fuel pressure. The fuel-discharge throttle valve was then adjusted to give the desired fuel flow, as measured by the rotameters. With these base conditions set, the sine-wave generator was started and the needle valve was adjusted to give the desired control-oil-pressure amplitudes at the minimum frequency. After allowing sufficient time for any transients in the system to disappear, the strip-chart-pressure-recorder drives and timer were started and the pressure variations were recorded. This procedure was repeated, keeping the same input amplitude but varying the frequency over a range from 0.3 to 8 cycles per second. Similar runs were made at different amplitudes, pump speeds, and fuel flows.

The possibility of introducing measuring errors due to the dynamic characteristics of the instruments was eliminated by interchanging the input and output recording circuits and making several frequency-response tests, as previously described.

RESULTS AND DISCUSSION

The steady-state characteristics of the pump are presented in figures 6 and 7. The relation between fuel pressure and control oil pressure taken at a fixed pump speed and outlet restriction area is shown in figure 6. The slope of the line is fixed by the servovalve design, however, and remains fixed regardless of the pump speed or outlet throttle. The effect of pump speed on fuel pressure at constant control oil pressure is given in figure 7. In the operating speed range of the pump (above 1400 rpm), the pump maintains practically constant fuel pressure, regardless of its speed or flow, by adjustment of the wobble-plate position.

Sample records of control-oil- and fuel-pressure traces, obtained during frequency-response tests, are shown in figure 8 for a frequency of 0.35 cycle per second and in figure 9 for a frequency of 3.6 cycles per second. Pulsations in the fuel pressure, believed due to vibration in the pump drive system, necessitated using faired curves to determine the fuel-pressure amplitudes and phase angles. Amplitudes were read from the pressure scales given on the charts and phase angles were determined from the synchronized timing marks on both charts. The pulsations in the fuel pressure and the decrease in wavelength with increasing frequency, which called for higher-than-recommended strip-chart speeds, rendered the phase-angle computations less accurate at higher frequencies. The proximity of the fuel-pressure trace of figure 9(a) to a true sine wave at a frequency of 3.6 cycles per second is shown in figure 10. It is to be noted that the superimposed vibration shown in figure 10 has the same amplitude (±5 lb/sq in.) and the same frequency (approximately 20 cps) as the vibration shown in figure 8(a).

All the runs were made at a base control oil pressure of 150 pounds per square inch, corresponding to a base fuel pressure of 340 pounds per square inch.

Typical frequency-response results are shown in figure 11. The ordinate of the amplitude-attenuation curve (fig. 11(a)) is the ratio of the measured fuel-pressure amplitude to the fuel-pressure change that is obtained in the steady state $(\omega = 0)$ for a change in control oil pressure equal to the control-oil-pressure amplitude, as given by a calibration curve, such as figure 6. (See equation (1).) The amplitude points of figure 11(a) follow the path of a first-order lag system with a time constant τ of 0.039 second and a corner frequency f_c (frequency at which the system attenuates to 0.707, or $f_c = \frac{1}{2\pi\tau}$) of 4.1 cycles per second; however, the phase-angle points of figure 11(b) deviate somewhat from a first-order system. It is known that the phase-angle measurements are less accurate than the amplitude-attenuation measurements. It should be noted, however, that the data points of figure 11(b) almost consistently fall below the first-order curve. It can be analytically shown that if the pump is a second-order system with a transfer function of the type $\frac{1}{(1+\tau s)(1+a\tau s)}$, then for small values of the constant a the amplitude curve follows very closely that of a first-order system, but the phase shift increases appreciably over that of a first-order system. The amplitude-response and phase-angle curves for a value of $a = 0.1$ are shown as dashed lines in figure 11.

The dynamic-linearity range of the pump was investigated by conducting frequency-response tests at the same base conditions but with input control-oil-pressure amplitudes of 20, 30, and 40 pounds per square inch. The results of these tests are presented in figures 12(a) and 12(b) for a pump speed of 1750 rpm. The frequency responses shown in these figures do not vary appreciably with amplitude; that is, the variation of corner frequency is slight and random and hence it is concluded that the pump remains linear for input amplitudes up to 40 pounds per square inch.

The effect of pump speed on frequency response is shown in figures 13(a) and 13(b), where results are shown for pump speeds of 1750 and 2500 rpm, covering a range of amplitudes. From figure 13(a) it is evident that increasing the speed increased the corner frequency from 4.6 cycles per second at 1750 rpm to 8.2 cycles per second at 2500 rpm. This increase in corner frequency is equivalent to a decrease in time constant from 0.034 to 0.0195 second. The faster response at higher speed is due to the smaller change in wobble-plate position necessary to achieve a given change in flow at the higher speed.

The decrease in phase shift accompanying the increase in corner frequency with increasing speed is shown in figure 13(b). For example, at a frequency of 4 cycles per second, the average phase shift at 1750 rpm is approximately 46°; whereas at 2500 rpm the phase shift is only 27°. The calculated phase-angle curves for first-order systems with corner frequencies of 4.6 and 8.2 (τ of 0.034 and 0.0195 sec), as determined from figure 13(a), are drawn together with the calculated curves for a second-order system of form $\frac{1}{(1+\tau s)(1+0.1\tau s)}$ with the same time constant as previously mentioned.

The effect of fuel flow on frequency response is shown in figures 14(a) and 14(b) for pump speeds of 1750 and 2500 rpm. At both speeds a change in fuel flow of almost 20 percent had no appreciable effect on the frequency response. This result is to be expected, inasmuch as conditions of different fuel flow at the same outlet pressure merely specify different base wobble-plate positions, which should not have any appreciable effect on the pump frequency response.

The effect of the fuel-pump response on the engine can be estimated by comparing attenuation curves. Figure 15 shows the attenuation curve for a jet engine, assuming a first-order system with a corner frequency of 0.16 cycle per second (time constant of 1 sec, reference 4), and also shows the attenuation curve for the fuel pump, assuming it to be a first-order system with a corner frequency of 6 cycles per second (time constant of 0.027 sec). The pump attenuates at a frequency much higher than the engine and, consequently, may be considered to respond instantly when compared with the engine.

The relatively negligible time constant of the pump, its wide linearity range, and the ease of obtaining sinusoidal pressure variations contribute to making this type of fuel pump an excellent device for providing sinusoidal fuel-flow inputs for conducting frequency-response tests on jet engines.

SUMMARY OF RESULTS

An axial-piston variable-stroke fuel pump was investigated by frequency-response methods employing a hydraulic sine-wave generator of varying frequency, amplitude, and base pressure as a means for obtaining a controlled pump input. The pump was found to approximate a first-order lag system with a time constant of the order of 0.04 second. Input amplitude and fuel flow did not have any effect on the pump response characteristics. Increasing the pump speed, however, decreased the time constant appreciably.

This type of fuel pump may be considered to have negligible lag when compared with current jet engines, and lends itself readily to producing a sinusoidal fuel pressure of varying amplitude and frequency for use as an input for frequency-response tests on jet engines.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, February 8, 1950.

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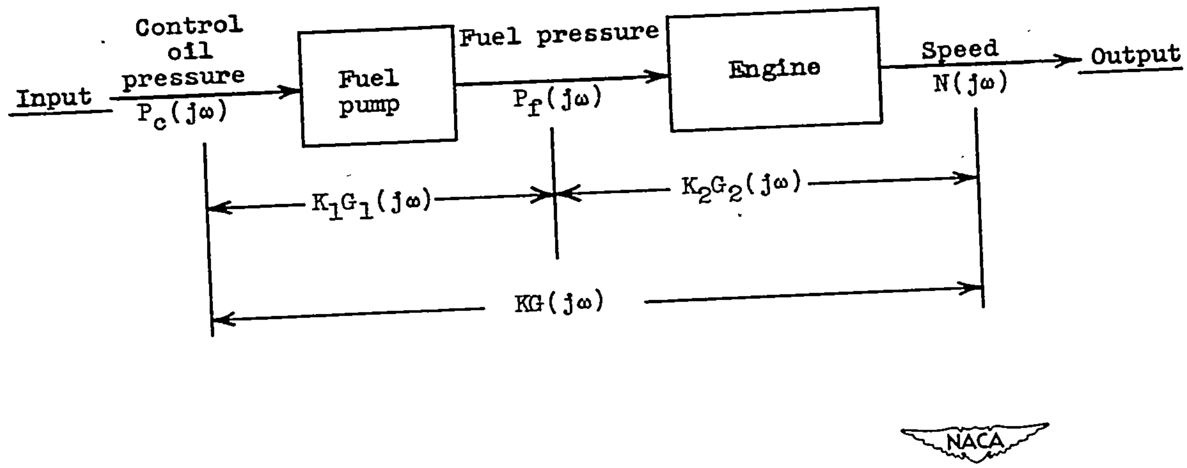
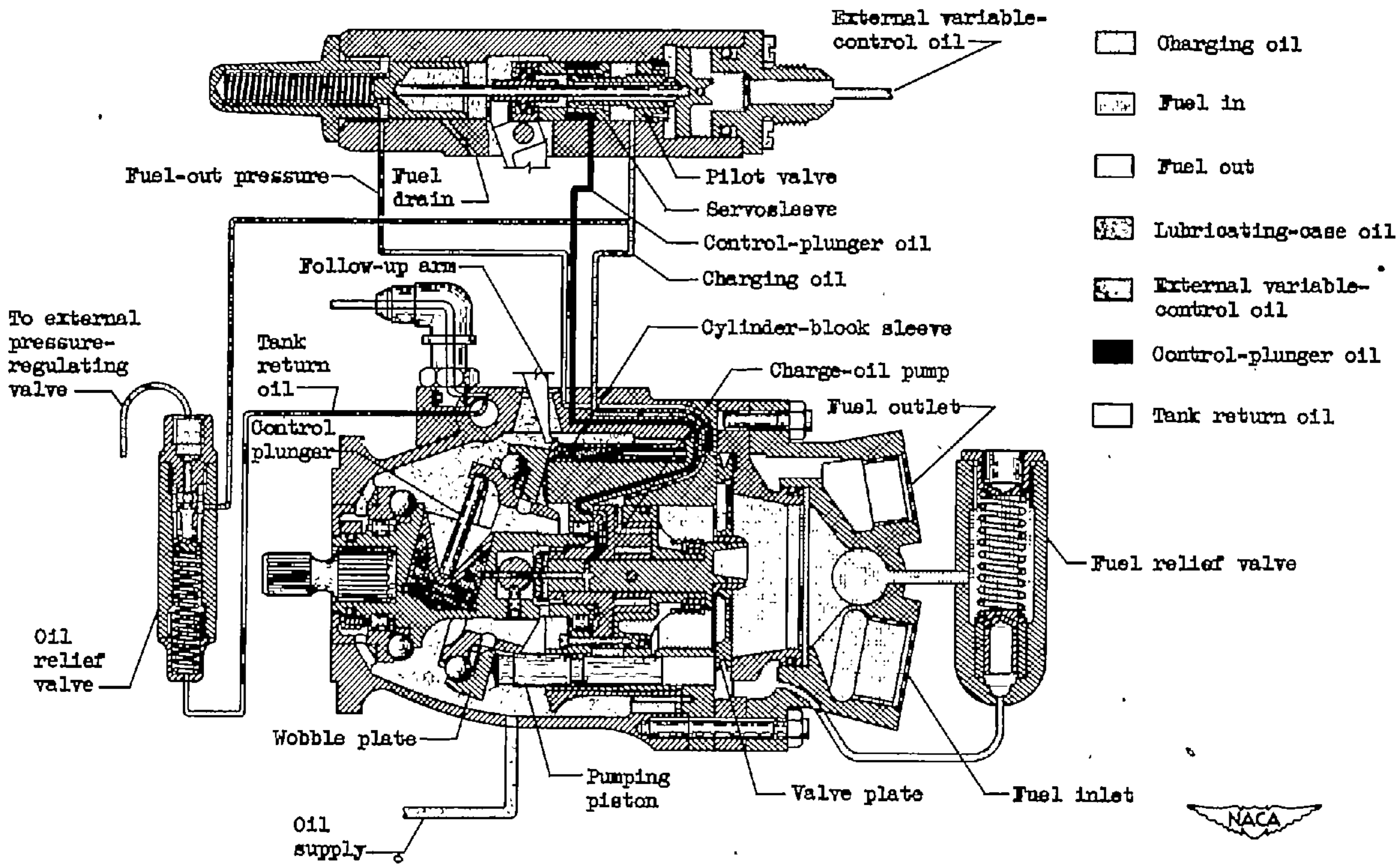
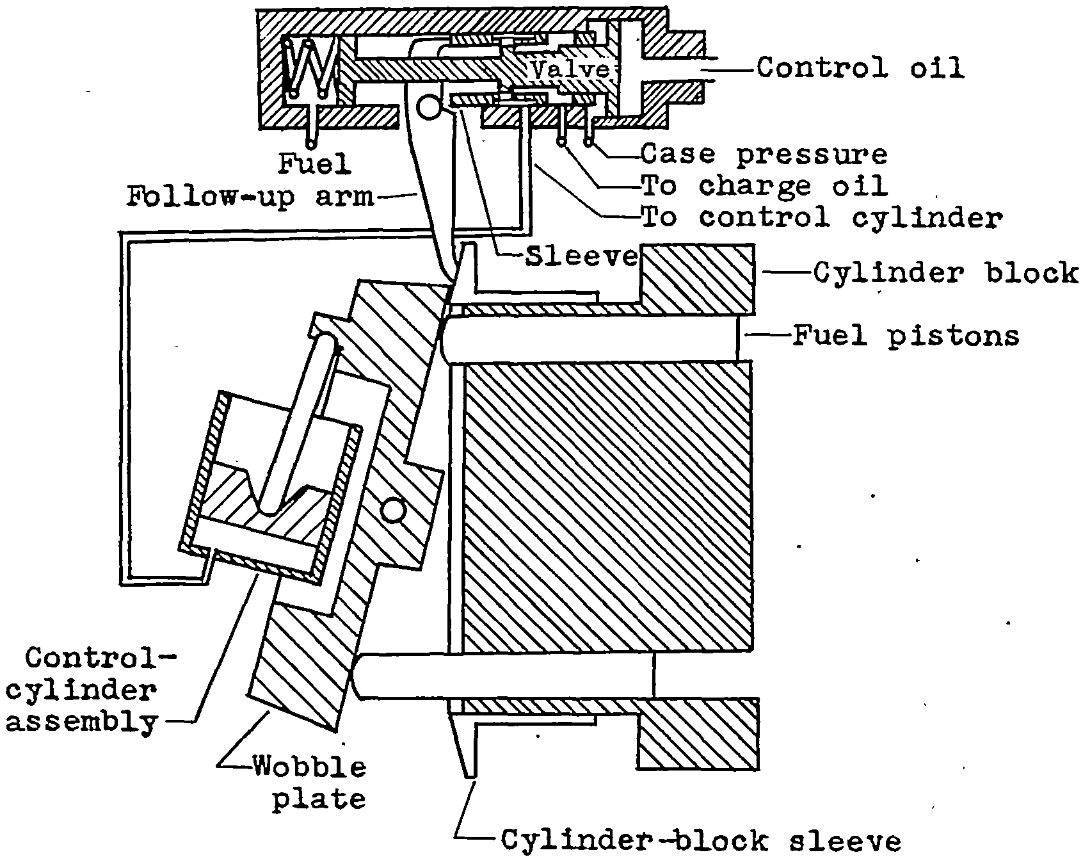


Figure 1. - Block diagram and transfer function of a fuel-pump and jet-engine combination.



(a) Schematic diagram of entire fuel-pump circuit.

Figure 2. - Schematic diagram of variable-stroke multipiston fuel pump with internal servocontrol system.



(b) Simplified symbolic diagram of servocontrol system.

Figure 2. - Concluded. Schematic diagram of variable-stroke multipiston fuel pump with internal servocontrol system.

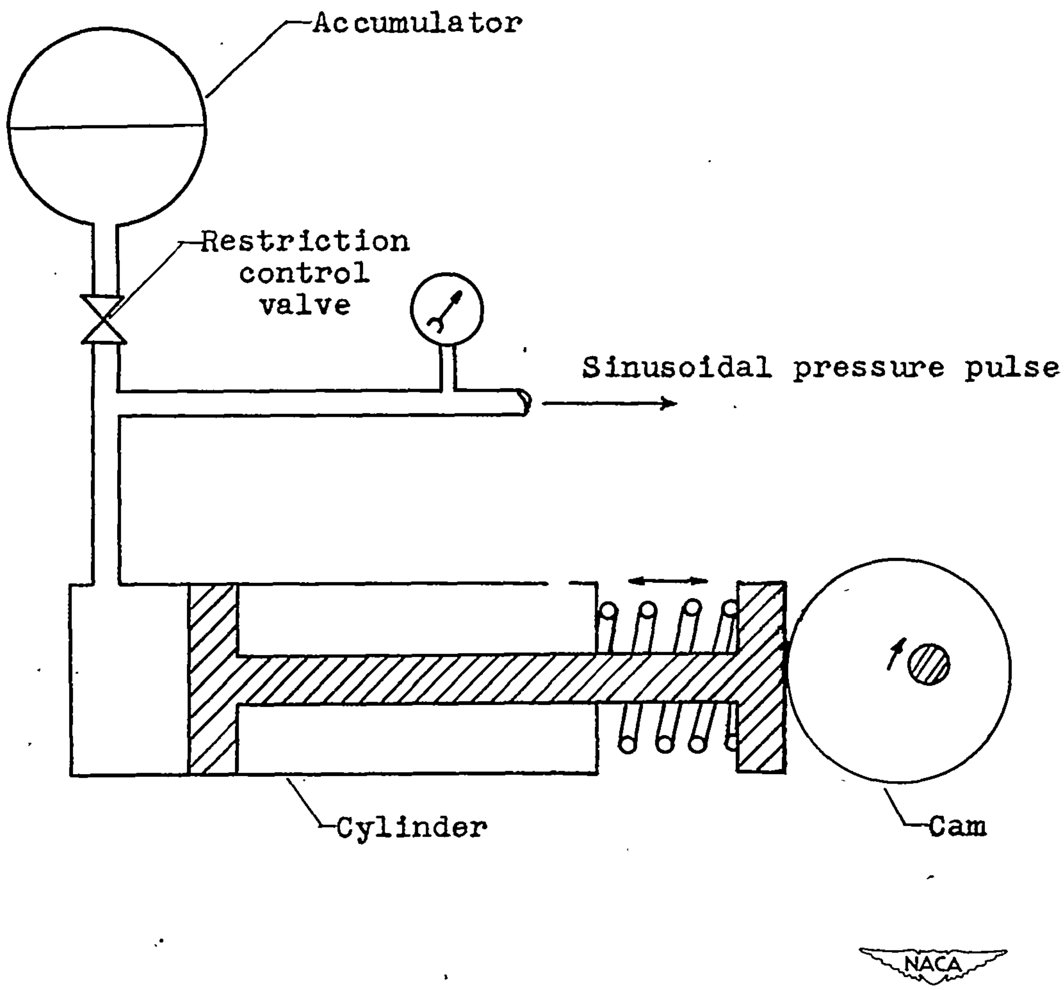


Figure 3. - Schematic of hydraulic sine-wave generator.

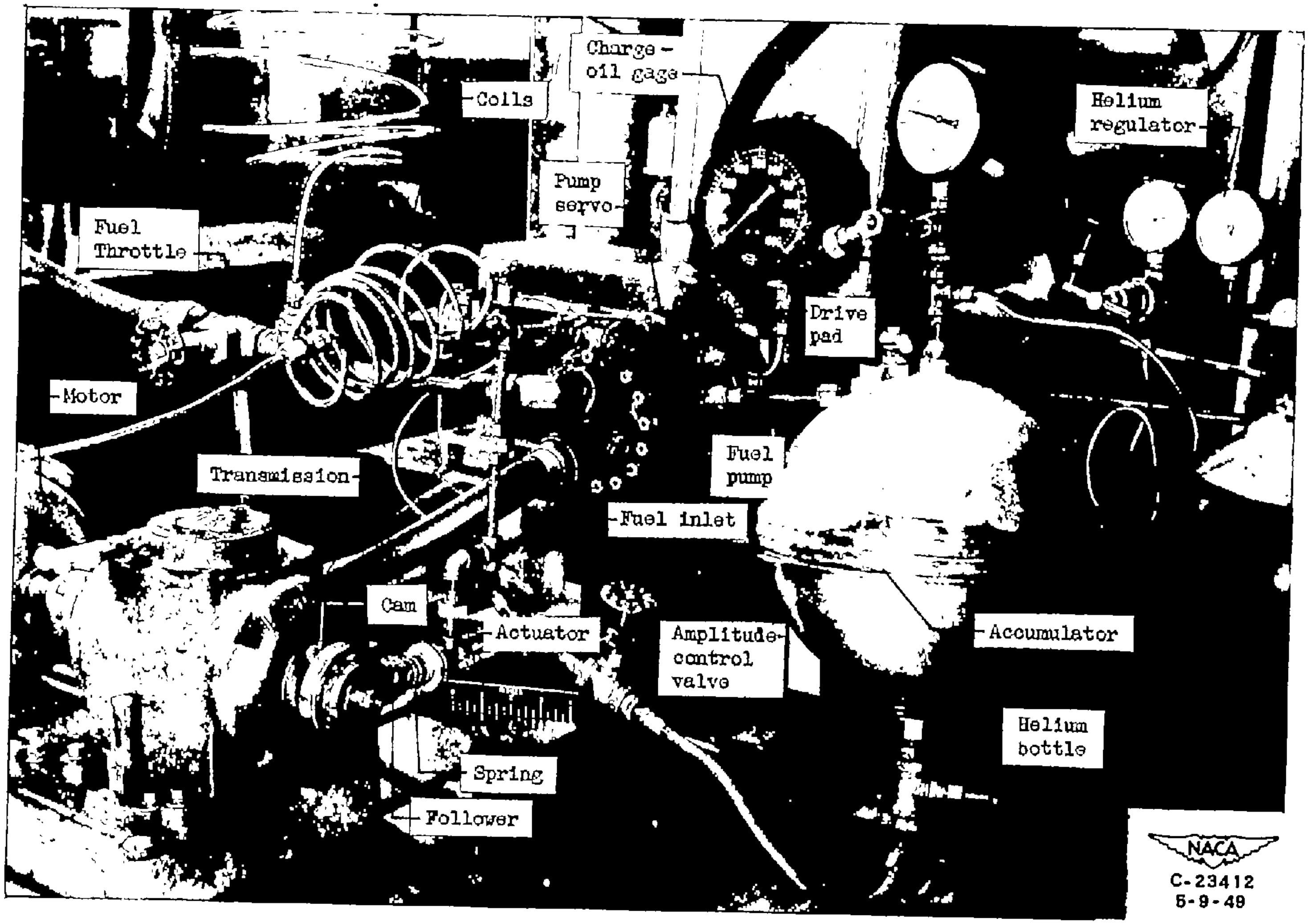


Figure 4. - Hydraulic sine-wave generator and fuel-pump frequency-response setup.

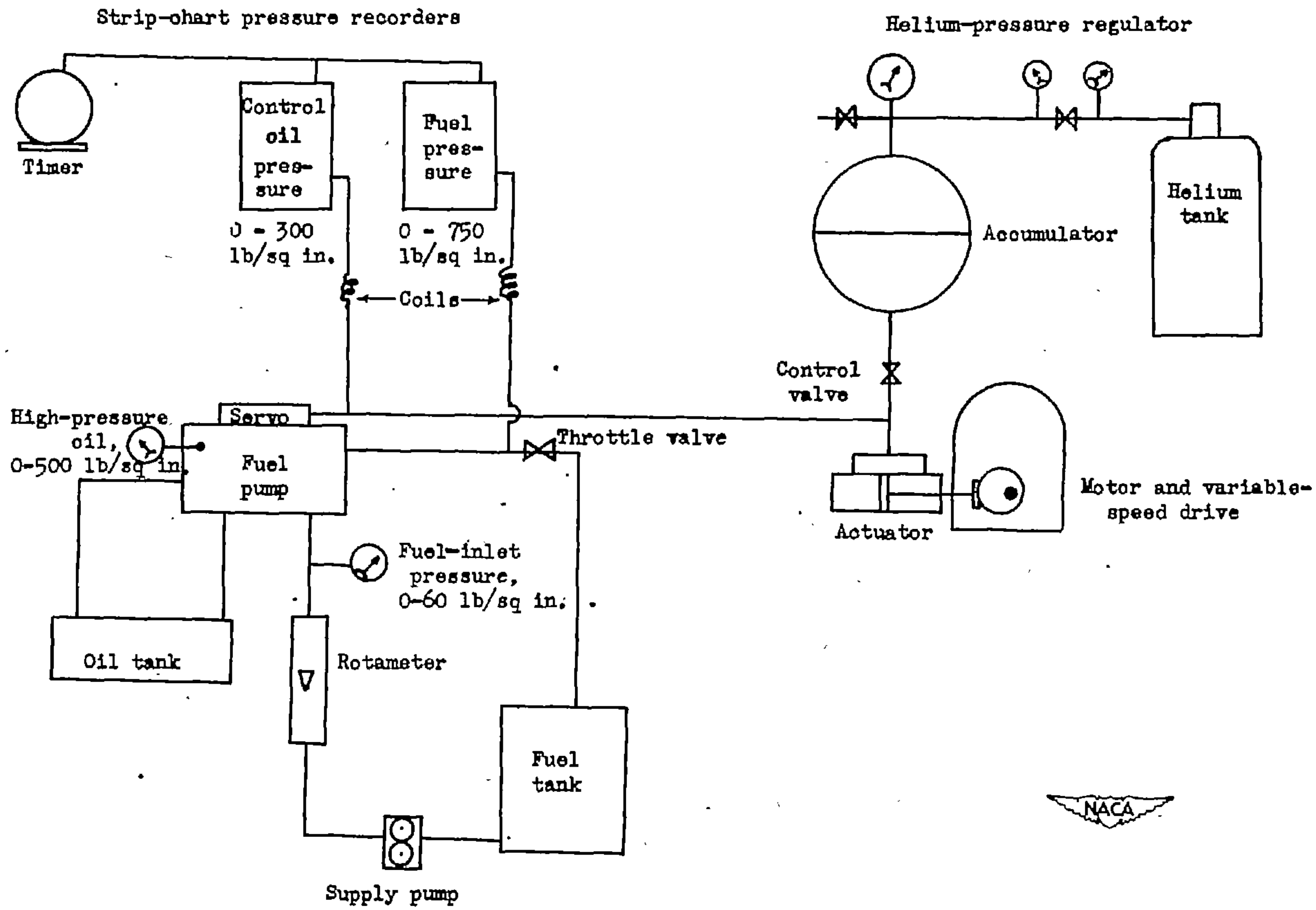


Figure 5. - Fuel-pump frequency-response setup.

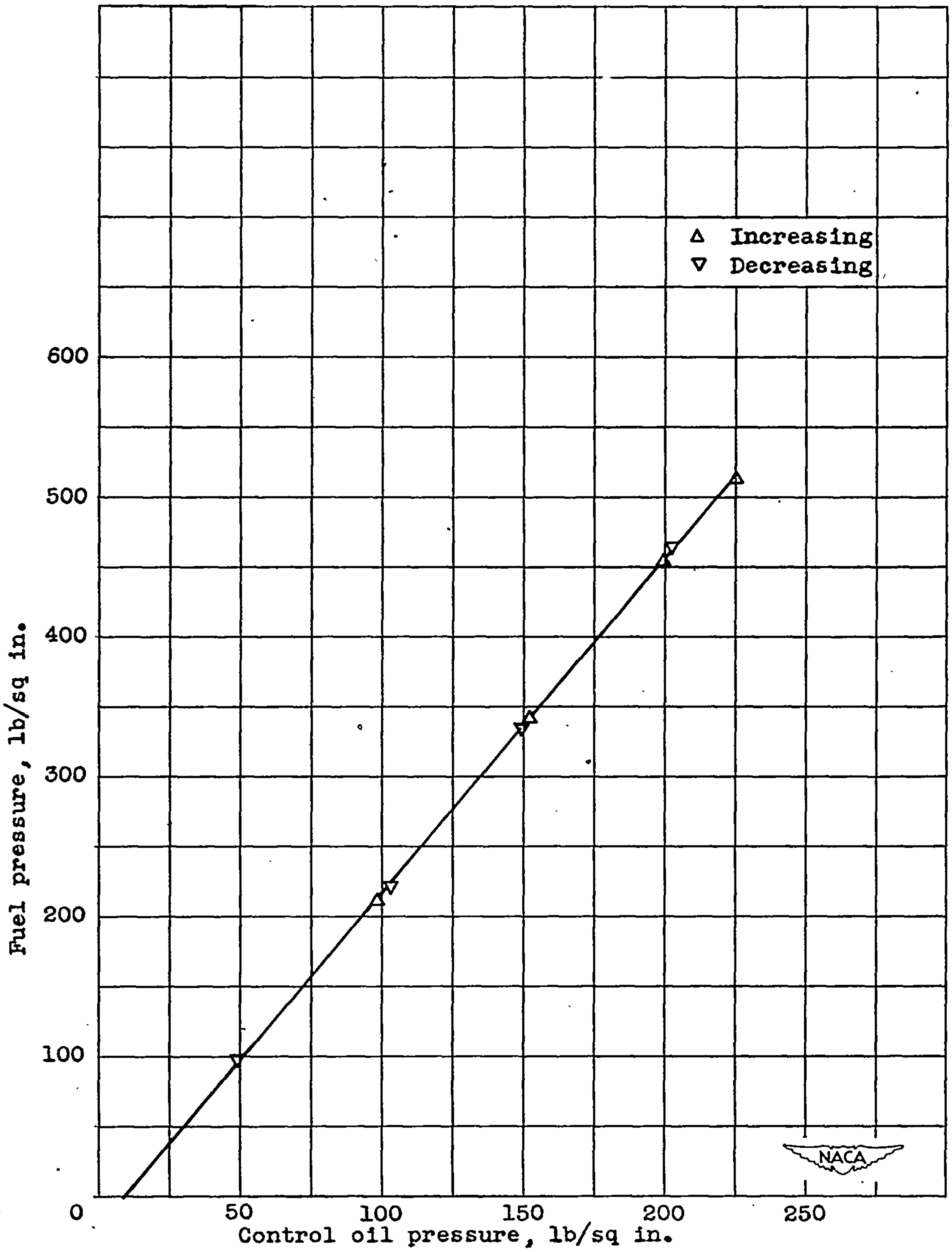


Figure 6. - Static calibration for variable-stroke axial-piston fuel pump.

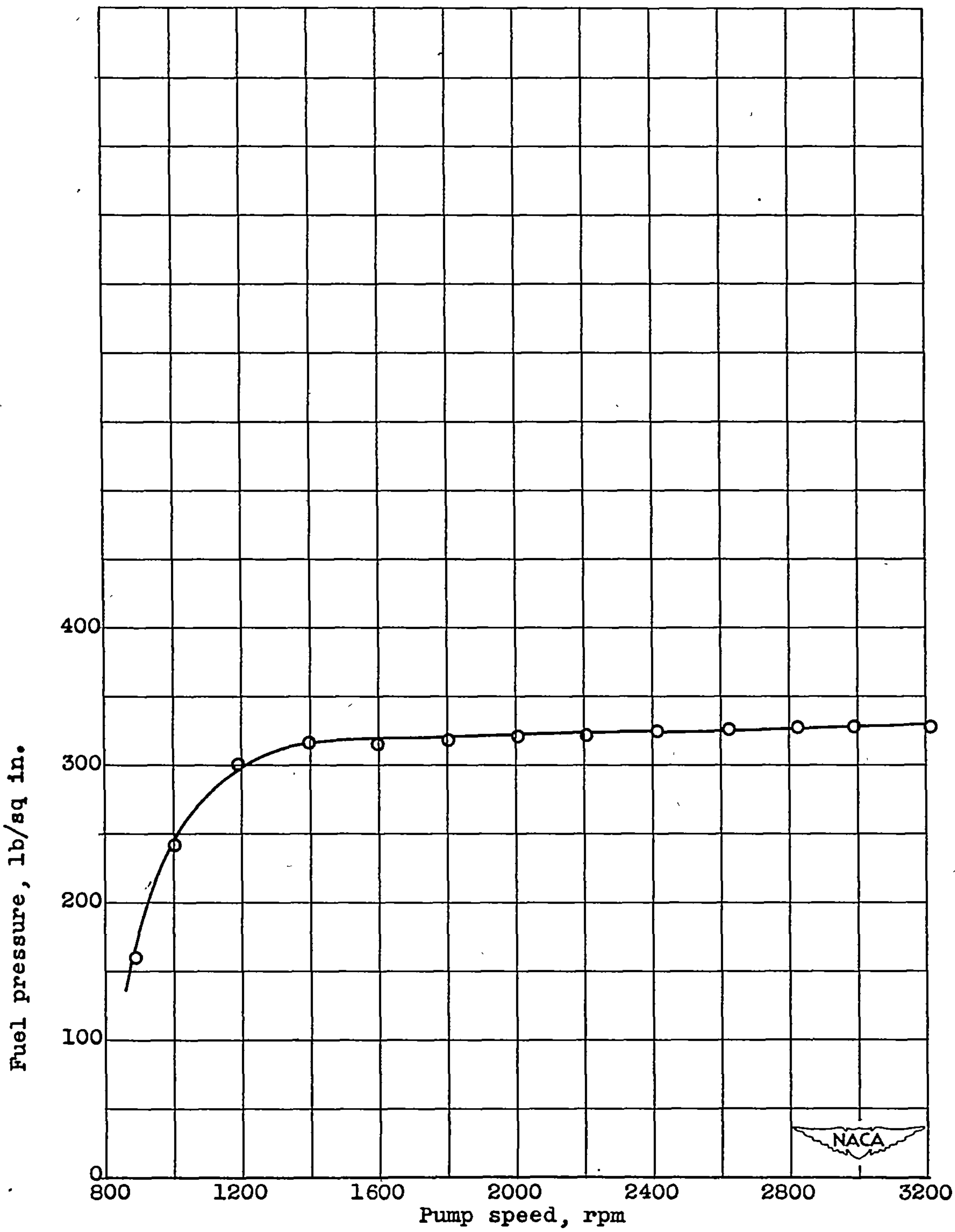
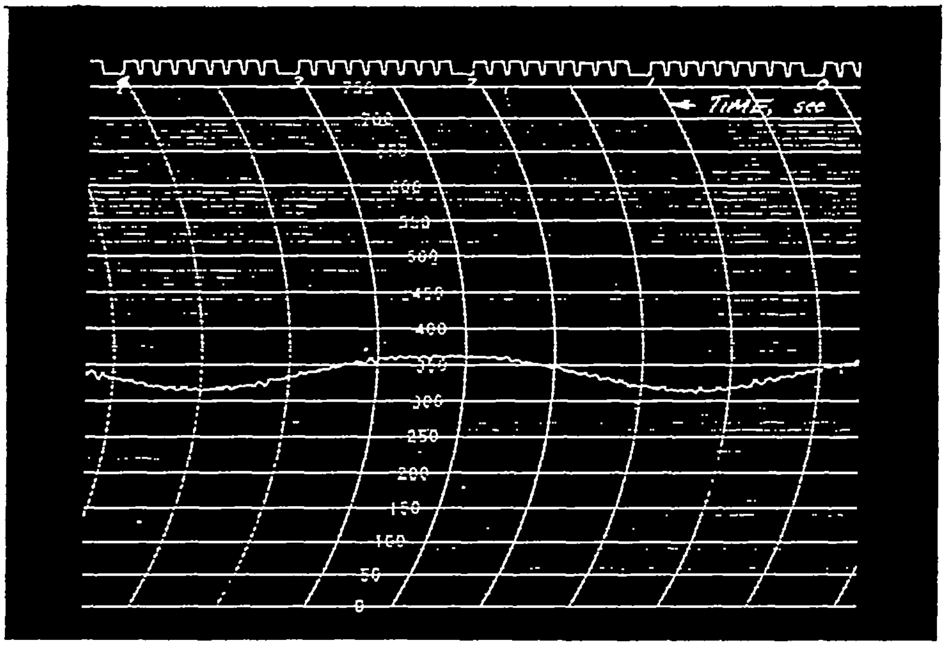
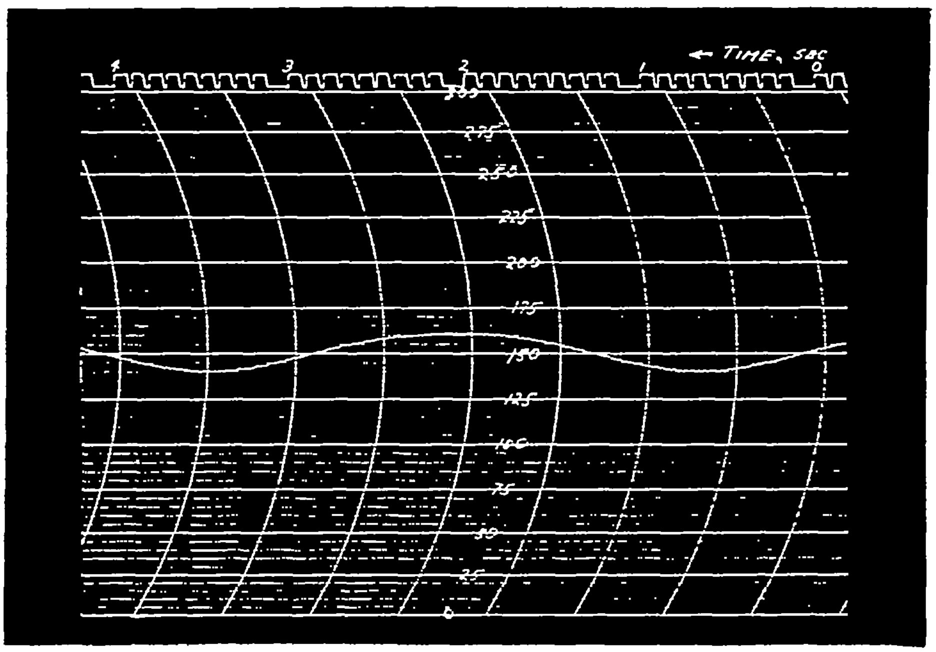


Figure 7. - Effect of pump speed on fuel pressure at fixed control oil pressure and outlet restriction.

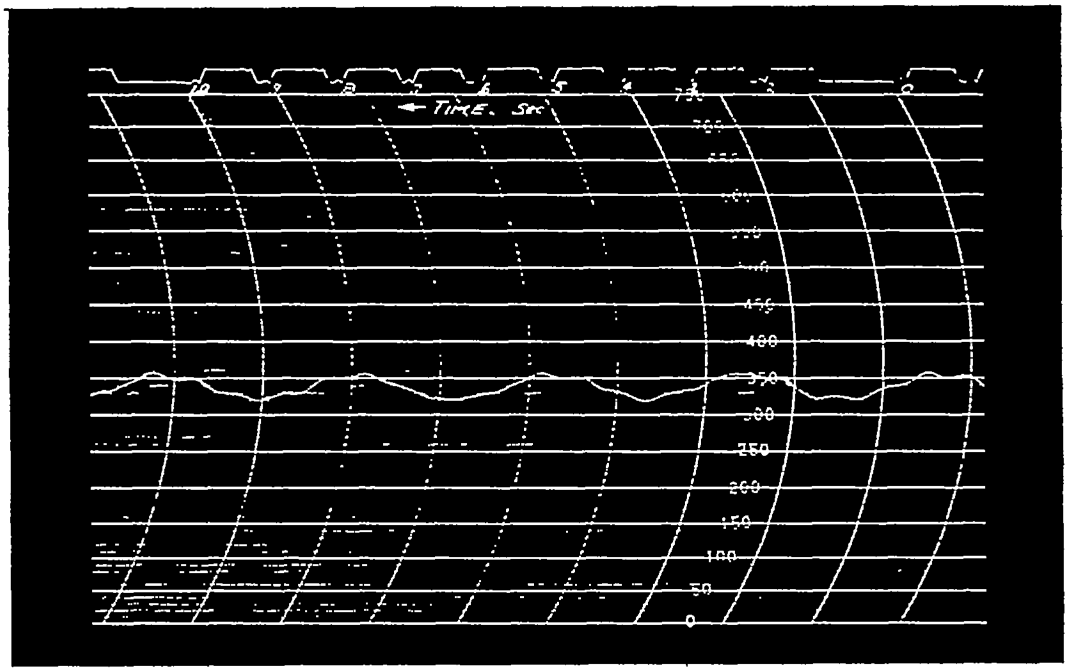


(a) Fuel pressure, pounds per square inch.

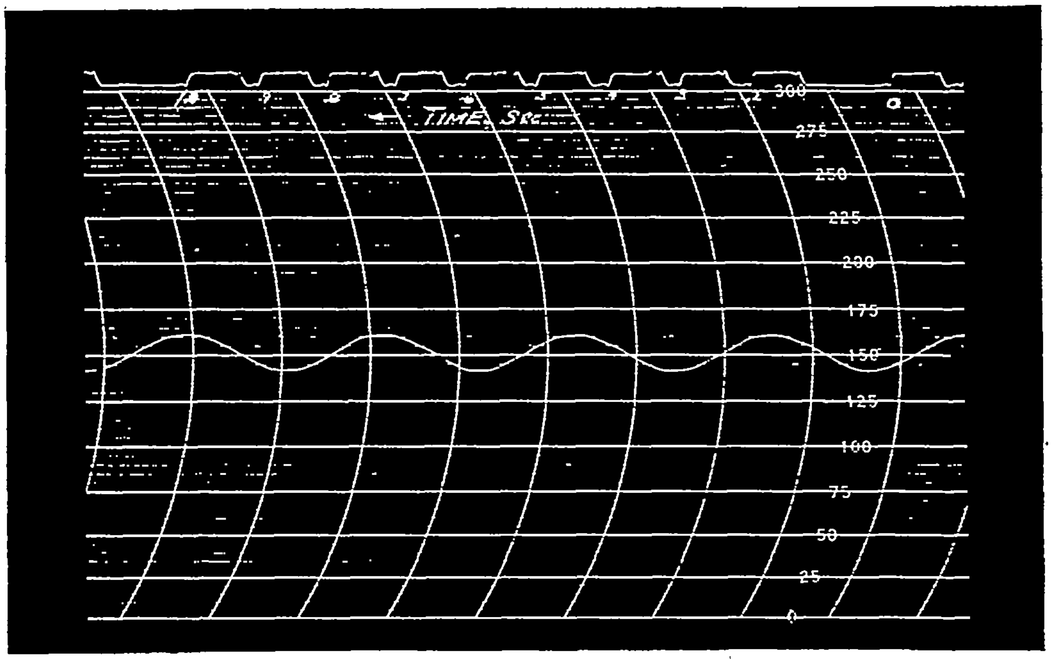


(b) Control oil pressure, pounds per square inch.

Figure 8. - Pressure traces at frequency of 0.35 cycle per second; pump speed, 1750 rpm.



(a) Fuel pressure, pounds per square inch.



(b) Control oil pressure, pounds per square inch.

Figure 9. - Pressure traces at frequency of 3.6 cycles per second; pump speed, 1750 rpm.

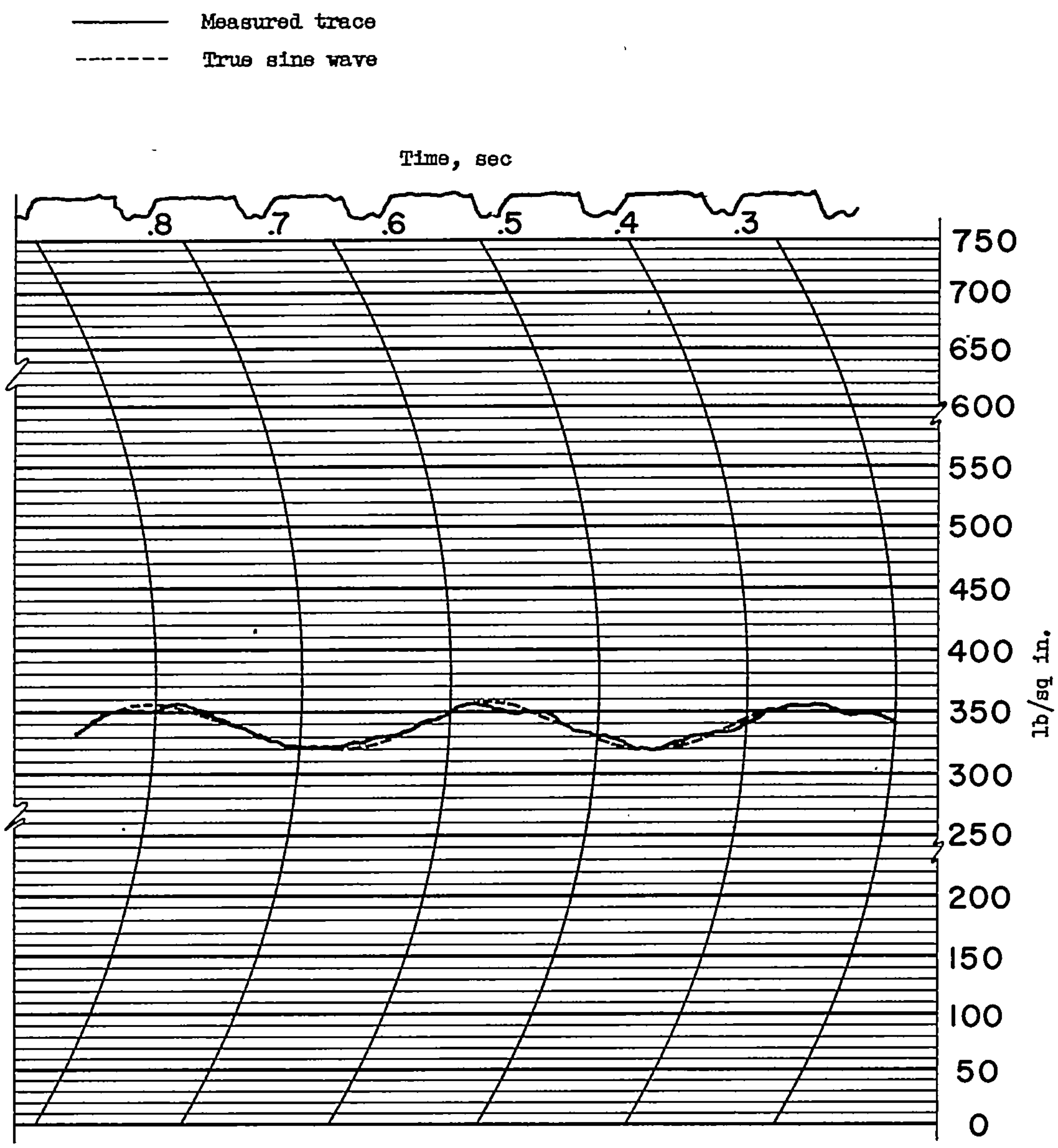


Figure 10. - Comparison of measured pressure trace at 3.6 cycles per second to true sine wave.

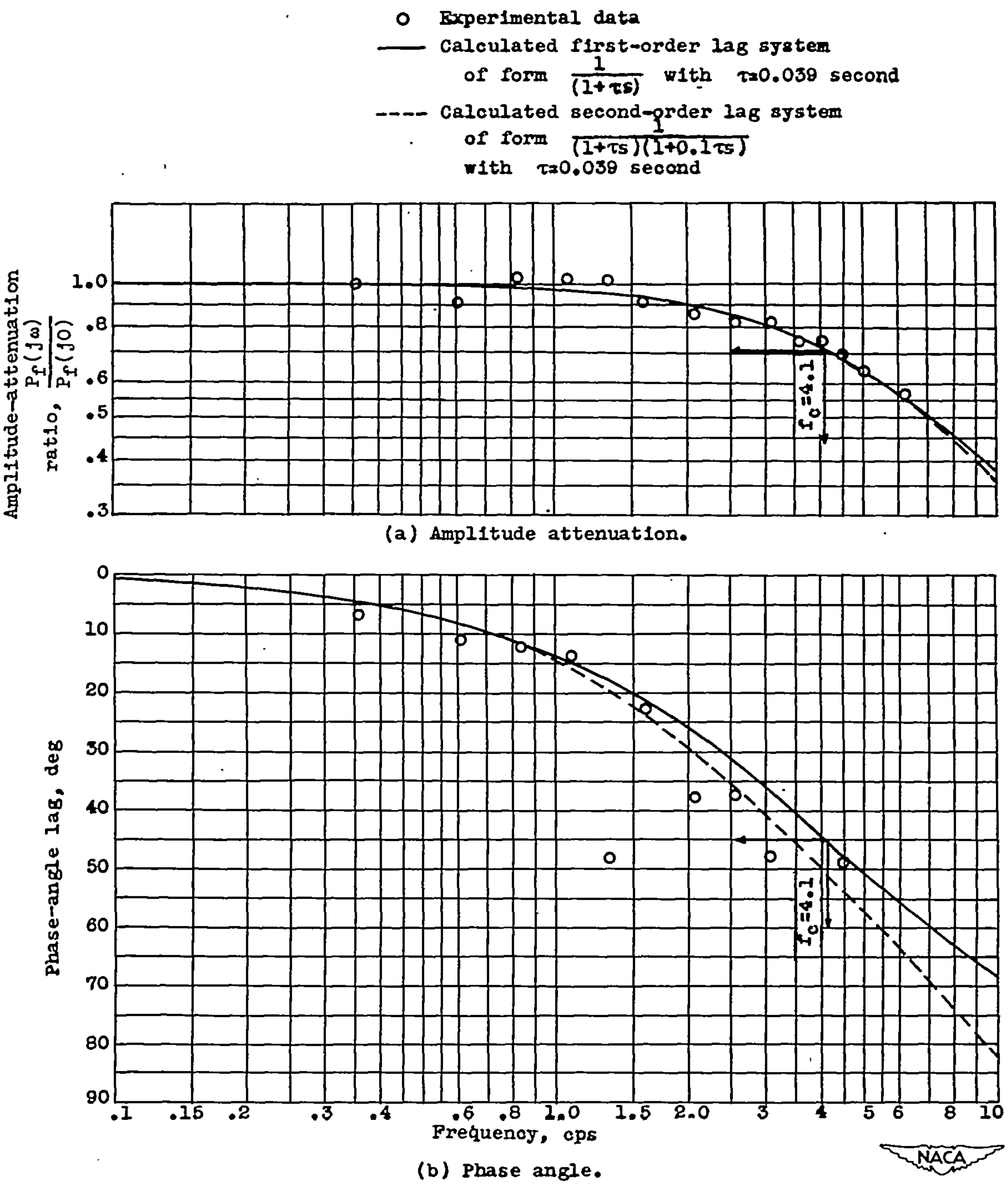
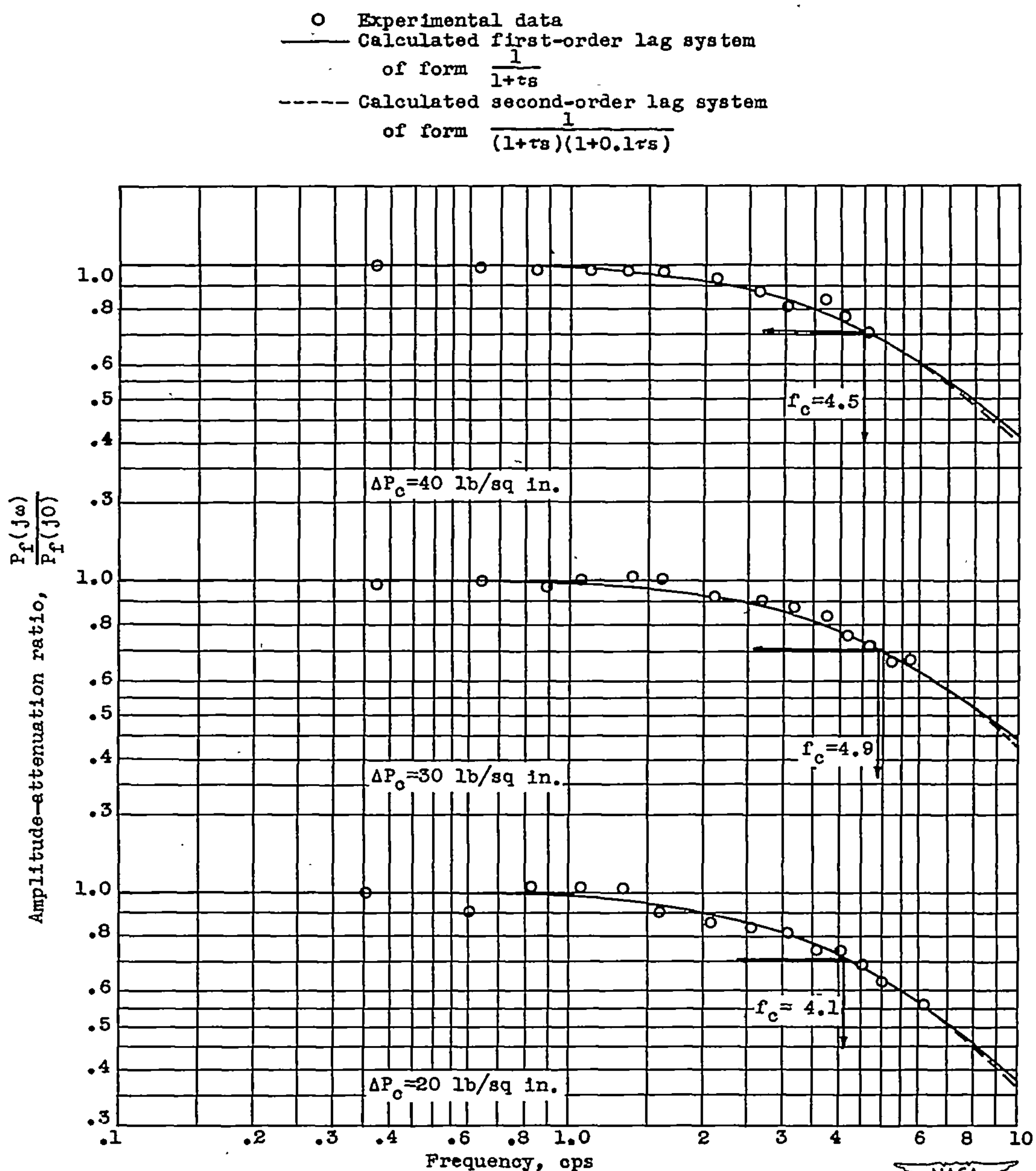
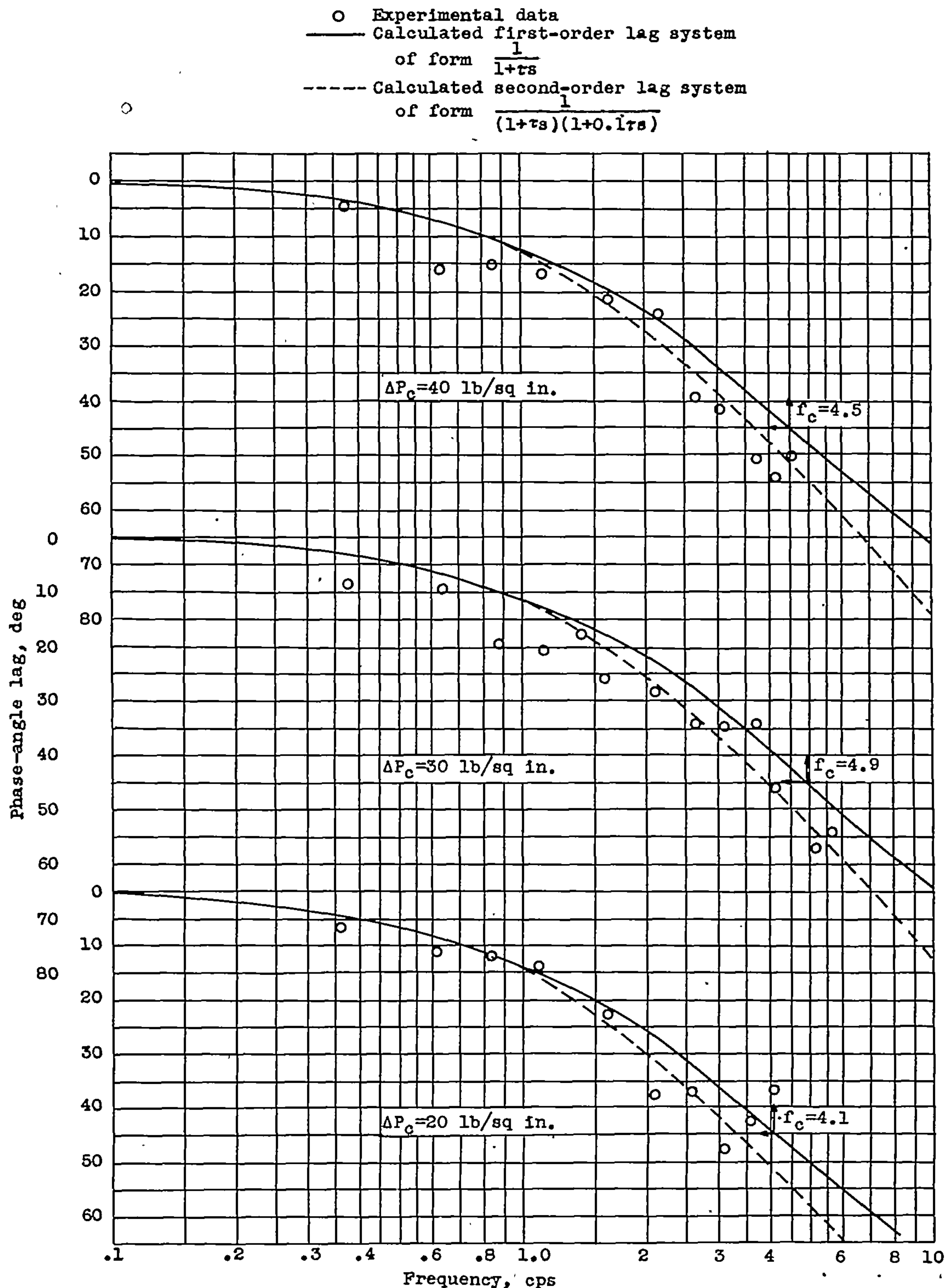


Figure 11. - Frequency response at pump speed of 1750 rpm, control-oil-pressure amplitude of 20 pounds per square inch, and base fuel flow of 2250 pounds per hour.



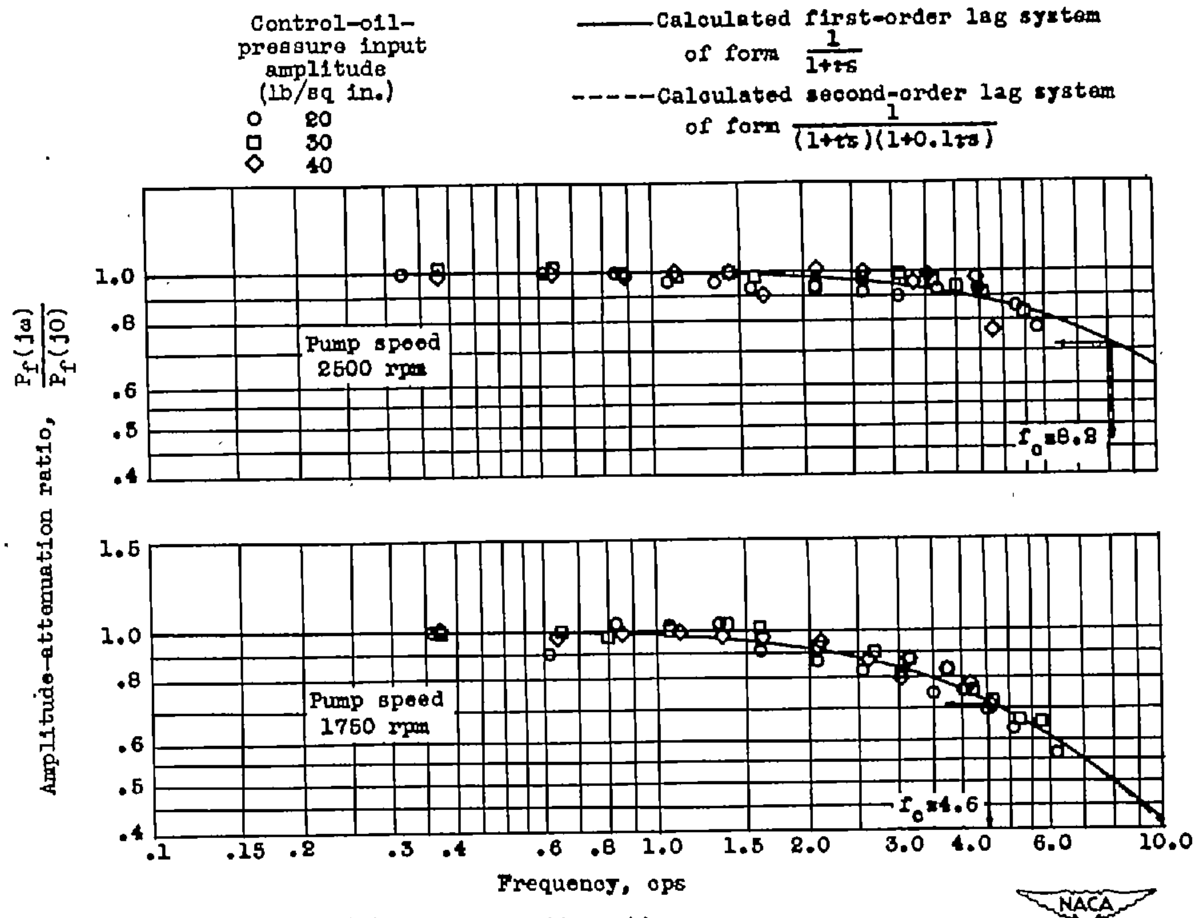
(a) Amplitude-attenuation curves.

Figure 12. - Effect of input amplitude at pump speed of 1750 rpm and base fuel flow of 2250 pounds per hour.



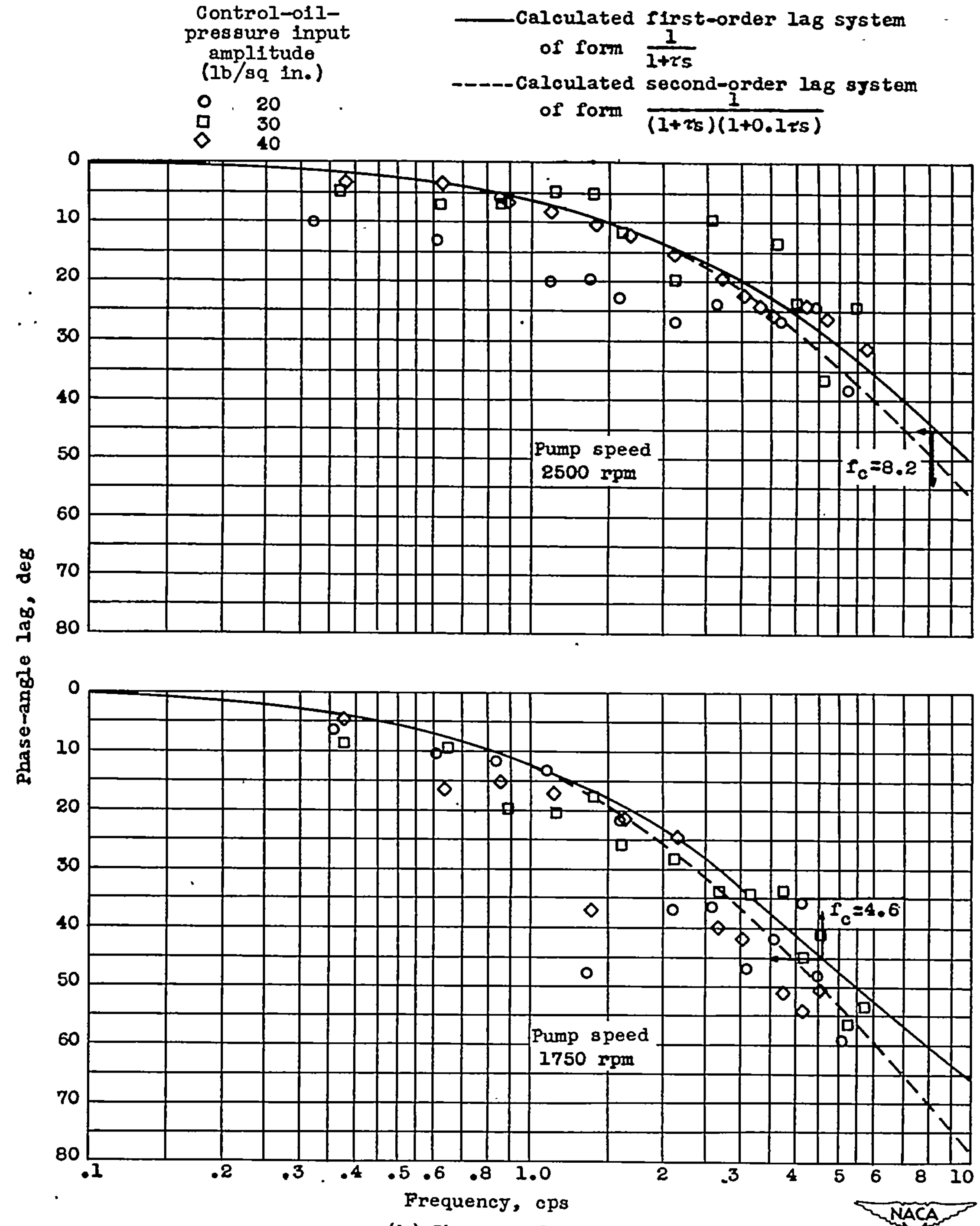
(b) Phase angle.

Figure 12. - Concluded. Effect of input amplitude at pump speed of 1750 rpm and base fuel flow of 2250 pounds per hour.



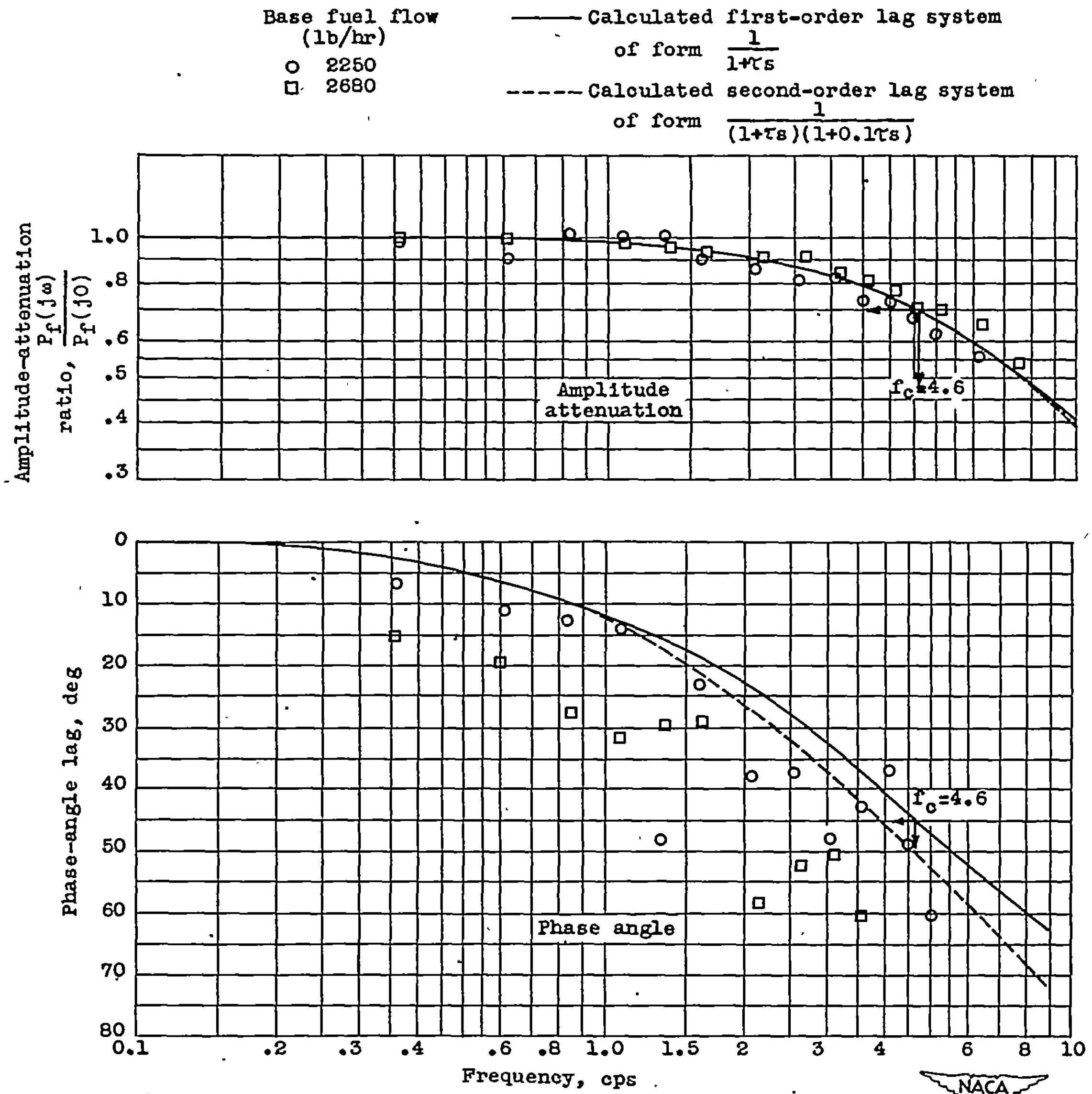
(a) Amplitude-attenuation curves.

Figure 13. – Effect of pump speed at base fuel flow of 2250 pounds per hour and control-oil-pressure amplitudes of 20, 30, and 40 pounds per square inch.



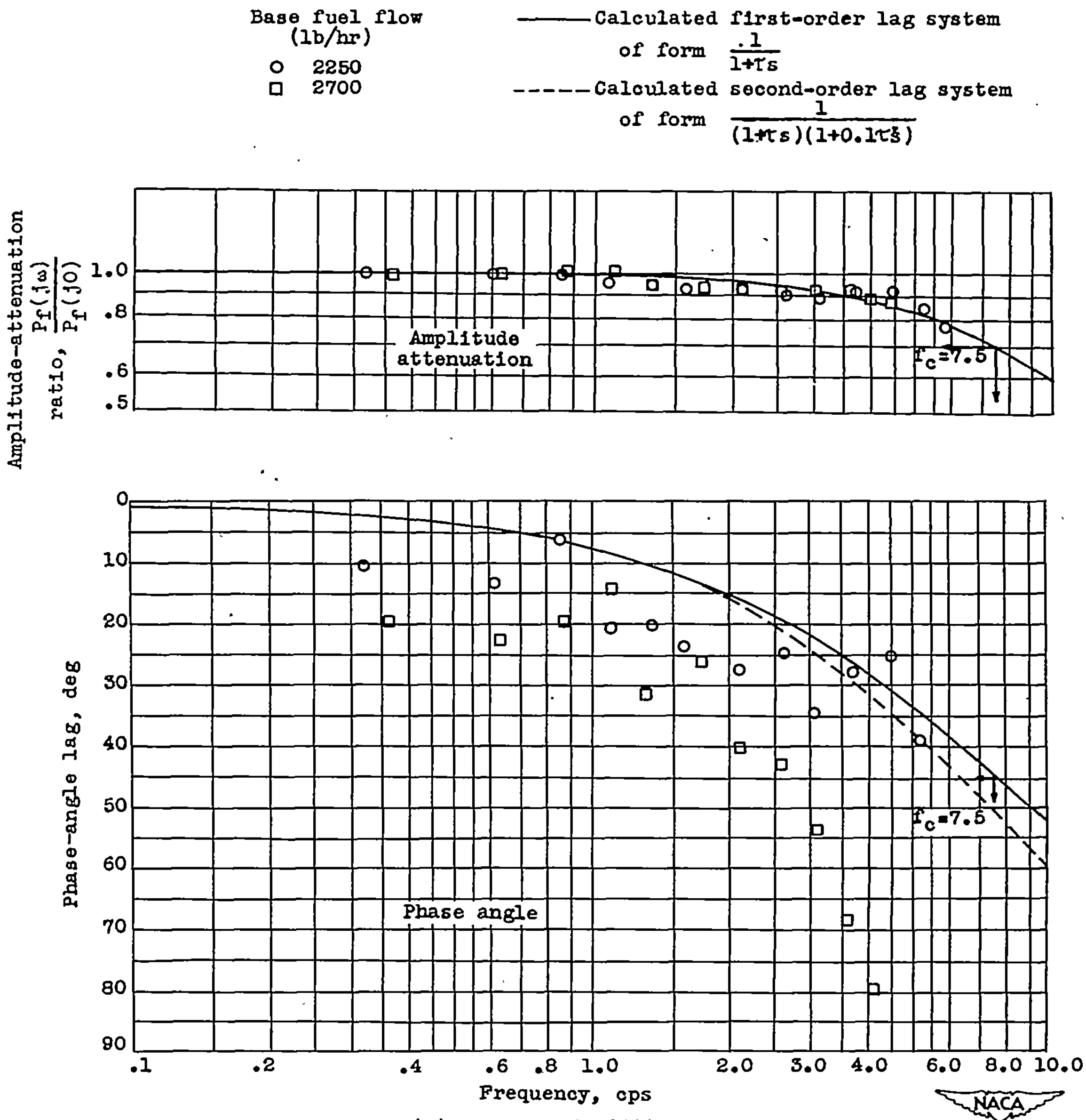
(b) Phase angle.

Figure 13. - Concluded. Effect of pump speed at base fuel flow of 2250 pounds per hour and control-oil-pressure amplitudes of 20, 30, and 40 pounds per square inch.



(a) Pump speed, 1750 rpm.

Figure 14. - Effect of fuel flow on frequency response at control-oil-pressure amplitude of 20 pounds per square inch.



(b) Pump speed, 2500 rpm.

Figure 14. - Concluded. Effect of fuel flow on frequency response at control-oil-pressure amplitude of 20 pounds per square inch.

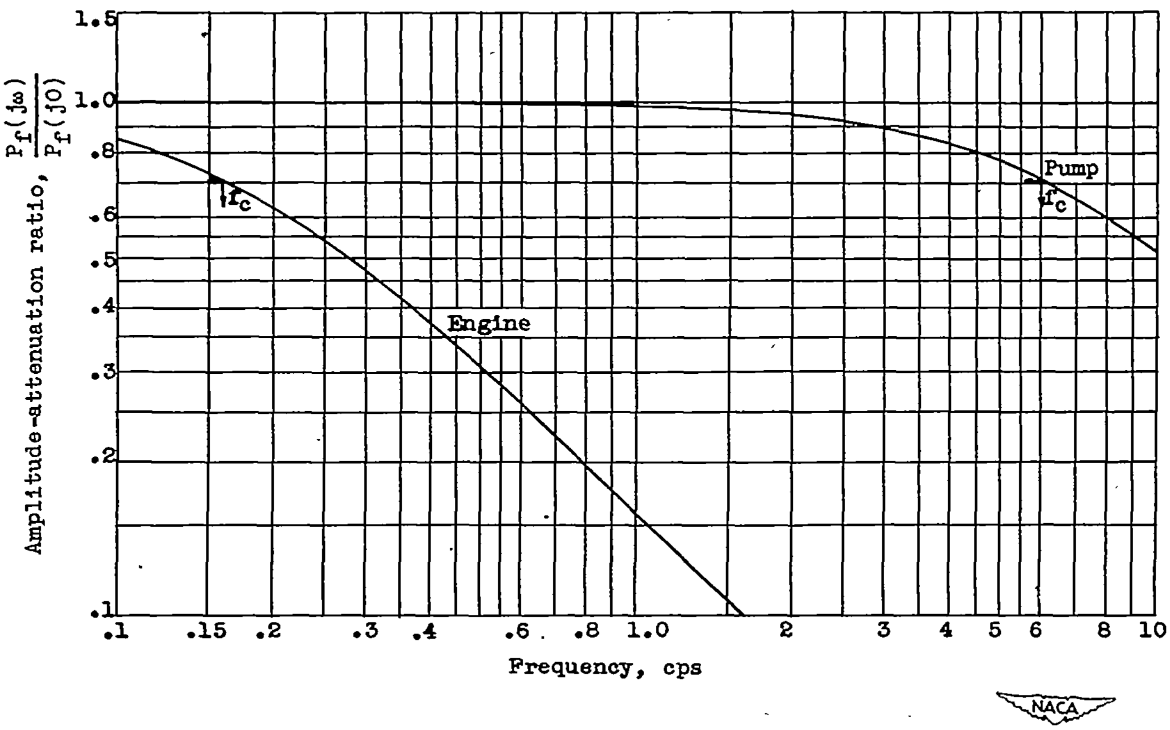


Figure 15. - Comparison of jet-engine and fuel-pump attenuation curves.

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